Find all non-isomorphic trees with 5 vertices.

We know that a tree (connected by definition) with 5 vertices has to have 4 edges. And that any graph with 4 edges would have a Total Degree (TD) of 8.

So our problem becomes finding a way for the TD of a tree with 5 vertices to be 8, and where each vertex has deg \( \geq 1 \).

\[ 1, 1, 1, 1, 4 \]

Reducing the deg of the last vertex by 1 and “giving” it to the neighboring vertex gives:

\[ 1, 1, 1, 2, 3 \]

Again, reducing the deg of the last vertex by 1 and “giving” it to a vertex with deg=1 gives:

\[ 1, 1, 2, 2, 2 \]

Mathematically, there is no more that we can do. These are the three equivalence classes.

Would the “stars and bars” approach we discussed previously also work here?

The short answer is no.

We have 5 vertices, which are our “categories,” and this gives 4 “bars.” Each category has at least one “star” already, since we have vertices of deg \( \geq 1 \).

\[
\begin{array}{cccccc}
\text{vert 1} & | & \text{vert 2} & | & \text{vert 3} & | & \text{vert 4} & | & \text{vert 5} \\
* & | & * & | & * & | & * & | & *
\end{array}
\]

so we have to distribute three more “stars” in those n=5 categories, giving:

\[ C(5-1 + 3, 3) = C(7, 3) = \frac{7!}{4! \cdot 3!} = 35. \]

So, this is much more than the 3 we had above!

But, we didn’t account for the permutations of the degrees within each equivalence class. Basically, it doesn’t matter which vertex gets the extra “star.”

For example, there’s no difference between \( 1, 1, 4, 1, 1 \) and \( 1, 1, 1, 1, 4 \). So there are 5 possible vertices with deg=4.

In \( 1, 1, 2, 3 \) there are \( 5 \times 4 = 20 \) possible configurations for finding vertices of degree 2 and 3.

And finally, in \( 1, 1, 2, 2, 2 \) there are \( C(5,3) = 10 \) possible combinations of 5 vertices with deg=2.

If we sum the possibilities, we get \( 5 + 20 + 10 = 35 \), which is what we’d expect. “Stars and bars” over counts since it doesn’t treat the vertices as indistinguishable.